The Implied Private Company Pricing Line: Empirically Observing the Cost of Capital
COC = FCFF/P + G

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In this paper, we show that small privately held businesses are not priced according to the Capital Asset Pricing Model (CAPM) or Modern Portfolio Theory (MPT); outline the many highly problematic comparisons between publicly traded equity securities and small privately held businesses; and develop an Implied Private Company Pricing Line (IPCPL) based on market approach transactions in small privately held businesses as our means to eliminate these highly problematic comparisons and to use as an accurate starting point to develop a cost of capital for any privately held company.

Introduction

The current ''state of the art'' for small business appraisal has no unifying or agreed upon approach, which has resulted in the current mass confusion related to determining the appropriate cost of capital for small privately held businesses.[1]

We borrowed Figure 1 to illustrate the current state of confusion.[2]

The source of the confusion can be highlighted with an old economist joke:

One moonless night a policeman saw an economist looking for something by a light pole. The policeman asked him if he had lost something. The economist said, ''I lost my keys over there in that dark alley.'' The policeman asked him: ''Then why are you looking way over here by the light pole?'' The economist responded, ''The light is much better here.''

The state-of-the-art cost of capital approaches (Duff & Phelps and Morningstar) have recommended the straight line, while some believe in the curve extrapolation—a cost of equity difference of as much as 23%. Many believe that size is a factor but stop well short of a 35% size premium. In this paper, we make a case that our industry's current cost of capital derivation techniques are unfortunately analogous to the economist joke above.

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Indeed, current practices are so controversial and incoherent, we get results and comments like these:

Dr. Paglia (Pepperdine Private Cost of Capital Project):[3]

''In a galaxy far far away, where unicorns prance on the back of the Loch Ness monster and privately held companies have access to public equity markets, appraisers estimate cost of capital by …(using returns of publicly traded equity securities).''[4]

The courts also have not been too impressed with our collective efforts:

Gesoff v. IIC Industries:

''This court has also explained that we have been understandably suspicious of expert valuations offered at trial that incorporate subjective measures of company-specific risk premia, as subjective measures may easily be employed as a means to smuggle improper risk assumptions into the discount rate so as to affect dramatically the expert's ultimate opinion on value.''[5]

These problems are due to the following insurmountable issues related to our state-of-the-art stock market return–based approaches, which have been debated and analyzed for years, and, if anything, these inherent problems have become more acute as time has passed.

- Company-specific risk: Current state-of-the-art data from recognized sources such as Morningstar data and Duff & Phelps do not capture company-specific risk (CSR). The current state of the art implicitly assumes that the marginal business investor is fully diversified.[6]
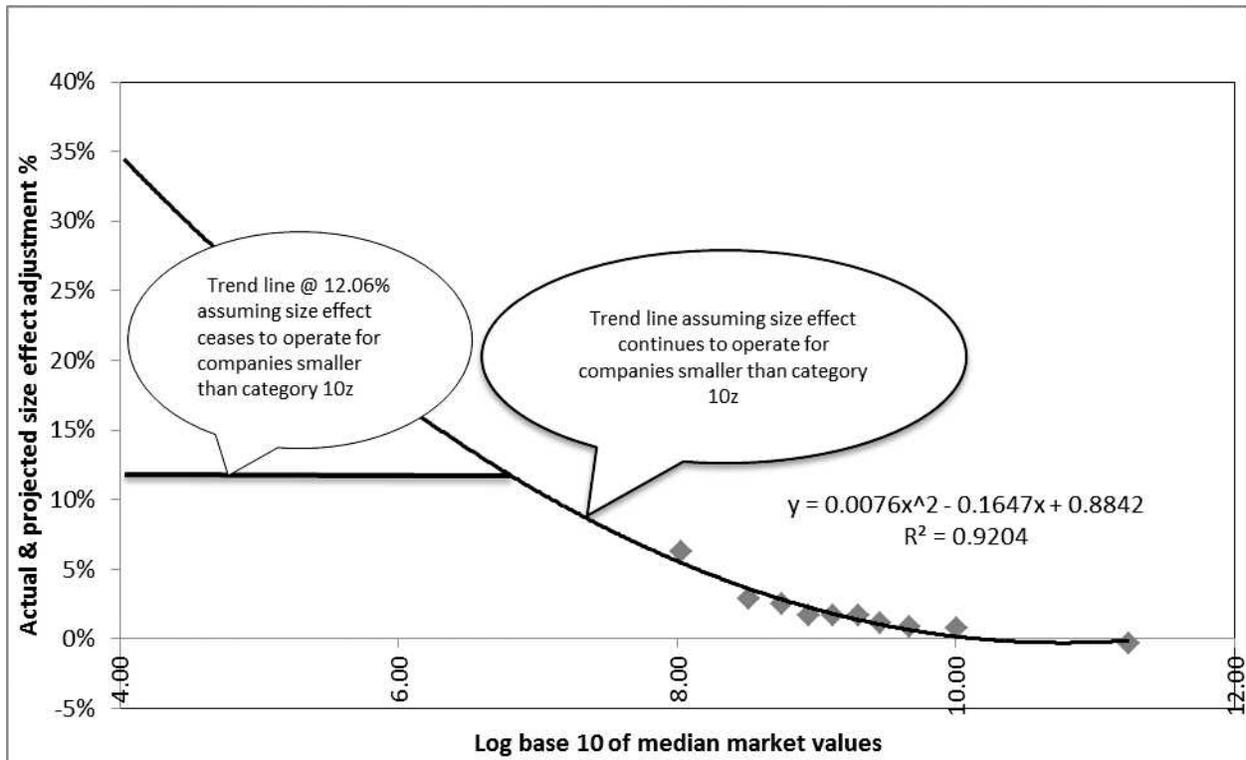
Figure 1
Size Effect Adjustment Percent

- Transaction cost differences: Total transaction costs for acquiring publicly traded equity securities related to sales commissions, valuation, and legal and accounting fees are approximately 0.5%. These same costs for acquiring a typical small privately held company are approximately 15%.
- Liquidity differences: Publicly traded stocks are highly liquid, requiring only a few computer key strokes on most any weekday to get cash in three business days. The current state of the art derives their returns from these liquid markets, and, as a consequence, the returns they extrapolate assume this extremely high liquidity. Even controlling interests in private firms are not nearly as liquid. For example, a business owner generally must incur very substantial up-front costs related to broker, appraiser, accounting, and attorney fees. In addition, due to limited liquidity, the seller may conservatively expect to sell her business in six to eighteen months, all the while hoping that the business is still worth as much or more at the time of the transaction. Compare this to selling 100 shares of publicly traded stock in a matter of seconds.
- Alleged ''small'' stock premium: The ''small'' stock premium is both controversial and highly complex. If one adopts the intertemporal flaw of Capital Asset Pricing Model (CAPM) as demonstrated by the Fama

French Three Factor Model, the current state of the art of using size percentiles will yield unreliable results. Please see Appendix B for more information on this vital topic.
- Taxes: Given that most appraisal assignments deal with a pass-through entity that pays no federal income taxes, should appraisers use a C-Corp income tax rate to remain true to CAPM/modern portfolio theory (MPT)? Alternatively, should appraisers use the Fannon Model, the Van Fleet Model, or the Treharne Model?
- Leverage: The current state of the art requires estimates of the percentage of debt to total capital, market interest rates, and re-levered beta estimates, all of which result in frequent errors.

As a consequence of these types of pitfalls, appraisers can only be sure to avoid these fatal flaws by applying the completed transaction (aka market) approach. With this approach, if the sample size of completed transactions is sufficiently large and comparable in terms of business, size, and margins, the appraiser could simply take the *observed* multiple from the comparables and apply it to her subject company. Using this procedure completely eliminates the inherent problems of adjusting for liquidity, transaction costs, and leverage, as well as CSR.

The appraiser need not worry about highly theoretical issues of cost of capital (COC) and modern portfolio theory (MPT) comparability. By using this approach, appraisers avoid all of the combined imponderable, theoretical, and debatable effects of the list above by utilizing the clearing-price dynamic of the competitive give and take between buyers and sellers in the completed transactions.

As a consequence of these very attractive attributes, we developed the Implied Private Company Pricing Line (IPCPL). The IPCPL uses the competitive clearing-price resolution of these imponderables by converting the observed transactions to the cost of capital. This way, the entire list of the intractable flaws of using the so-called state of the art as a basis to value small privately held businesses is completely eliminated.

Publicly Traded Equity Securities vs. Small Privately Held Businesses

Dr. Paglia of The Pepperdine Private Cost of Capital Project asserts that comparing publicly traded equity securities to small privately held businesses is akin to valuation malpractice. Their axiom in this regard is that the two are hopelessly different because they raise capital in completely different capital markets. We believe this is an important observation. However, we believe that an even more important distinction is that the small privately held businesses and publicly traded securities are hopelessly different *assets*.

Investors can buy a publicly traded company's stock with significant uncorrelated risk for about $2,000 (say 100 shares at $20/share) and incur a commission of $6.00 (about 0.5% of the investment). In order for this investment to not violate MPT (about one-thirtieth of net worth), the investor need only have a net worth of $60,000 in order to be confident that her portfolio will bear no significant uncorrelated risk after the $2,000 purchase. Therefore, publicly traded securities, due to the extreme ease of diversification, have, in effect, zero company-specific risk on an ex ante basis. Furthermore, since publicly traded securities are continuously traded and subject to sophisticated long or short speculators and or arbitrage, investors can freeload on this due diligence and, consistent with efficient market theory, simply buy a diversified portfolio without incurring any due diligence costs. Therefore, the total costs related to the transaction, diversification, total liquidity, and due diligence are a mere 0.5% of the investment.

The major distinction with small privately held companies is that one cannot buy .00000001% of the company that is being listed by the business broker. The buyer must buy 100%—say, $1,000,000. If we apply the same MPT

diversification rule (one-thirtieth), the buyer is required to have a net worth of $30,000,000. Therefore, the vast majority of business buyers and sellers/owners are extremely undiversified. Furthermore, the asking price for a small privately held business, unlike the actively traded equity security, is not subject to the long and short speculators and arbitrageurs. Therefore, the marginal small privately held business buyer must incur very substantial due diligence fees in the form of valuation analysis, legal analysis, and accounting fees. In addition to these fees, the commission for the business broker is typically 10%. Therefore, we can reasonably estimate these total costs at approximately 12% to 14% compared to 0.5% for the publicly traded equity security.[7] However, even after considering this large difference, the typical buyer would still be almost completely undiversified and own an asset that may take a year or longer to convert to cash.

The modern portfolio theory advocate could claim that buyers and sellers could sell minority stock to passive investors in the small privately held company to accomplish diversification. However, we can observe, that for the most part, they do not. This could be due to any number of reasons, including some of the well-known problems of minority interests such as owner-operator conflicts of interests (converting profits to owner compensation, etc.) and *additional* substantial due diligence costs in the form of valuation, legal, and accounting fees.

The MPT advocate might then argue that several high-net-worth individuals, companies, or funds could come in and arbitrage, thereby competing away the pricing of uncorrelated diversifiable risk. Again, we see that this almost never happens. The due diligence costs are very high, and the liquidity is very low. Perhaps the owner-operator concept is vital in that having a manager that reports to shareholders creates significant monitoring costs that the owner-operator does not incur. Furthermore, the owner-operator's manager versus owner incentives are perfectly aligned, whereas the manager vs. shareholder incentives are imperfectly aligned.

The evidence with respect to these issues is that we see almost zero non-owner-operator, large-net-worth equity investors or passive shareholders in small firms. This fact suggests that although the typical owner-operator is highly undiversified, the costs of diversification (either by selling stock to numerous passive minority investors or by requiring the pricing of the implied imperfections associated with a non-owner-operator described above) are *higher* than the utility-based costs of entrepreneurs/owner-operators bearing company-specific risks.

Finally, the MPT advocate may argue that these highly undiversified owner-operators are risk neutral or different

than normal risk-averse investors.[8] However, we employ the research found in two recent papers that specifically address this potential issue. Both of these papers conclude that the entrepreneur is substantially risk averse. In ''Entrepreneurial Finance and Non-Diversifiable Risk,'' the article states the following:

> ''Entrepreneurial investment opportunities are often illiquid and non-tradable. Entrepreneurs cannot completely diversify away project-specific risks for reasons such as incentives and information asymmetry. Therefore, the standard law-of-one-price based valuation/capital structure paradigm in corporate finance cannot be directly applied to entrepreneurial finance. In addition to compensation for systematic risks, the entrepreneur also demands a sizable premium for bearing idiosyncratic risks, which increase with his risk aversion, his equilibrium inside ownership, and the project's idiosyncratic variance.''[9]

Also, in the ''Investment, Idiosyncratic Risk, and Ownership'' paper, the authors state:

> ''High-powered incentives may induce higher managerial effort, but they also expose managers to idiosyncratic risk. If managers are risk averse, they might underinvest when firm-specific uncertainty increases, leading to suboptimal investment decisions from the perspective of well-diversified shareholders. We empirically document that, when idiosyncratic risk rises, firm investment falls, and more so when managers own a larger fraction of the firm.''[10]

Development of the Implied Private Company Pricing Line (IPCPL)

In the land of the blind, the one-eyed man is king.

—Erasmus of Rotterdam, circa 1510

We begin with the Gordon Growth Model, which is both axiomatic and fundamental to modern finance and is the core basis of the IPCPL:

$$P = FCFF/(COC - G)$$

where:

P equals price paid/FMV (total equity plus interest-bearing debt);

FCFF equals the annual free cash flow to the firm, unlevered;

COC equals the firm's cost of capital; and

G equals the annual perpetually stable growth rate of FCFF.

We can rearrange the terms algebraically to the following formula:

$$COC = FCFF/P + G$$

The elegance of this model is that we if we know FCFF, P, and G, then COC can be derived. Stated differently, the state-of-the-art or build-up method (BUM), with its long list of problematic theories, need not apply; we can empirically ''observe'' the cost of capital.

Our Data Set and Conclusions

For our study, we incorporated a large data set of 830 qualifying transactions listed in Pratt's Stats and BIZCOMPS in a multiple regression model that solved for the normalized ratio of operating income divided by P.[11]

Our experience with this approach is that people understand and trust the core model, $COC = FCFF/P + G$. They also find the model conclusions both logical and consistent with their experience. Therefore, before we get into the details of our methodology, we present our fundamental methodology and conclusions, so that one can assess the value of our research and ultimate conclusions before focusing on secondary issues related to model details and data conversion methodology.

Based upon our multiple regression model and our data set of 830 transactions, we derived a normalized operating income multiple for our median-sized ($4 million in revenue) privately held company. We define the ''normalized'' operating income multiple generally as the operating income (adjusted for market-based owner compensation) multiple based on operating income that is stable.[12]

Our normalized conclusion for this size indication was a multiple of 4.51, consistent with our, as well as others', experiences. Perhaps more importantly, our model concluded a strong and statistically significant positive correlation between size and the normalized operating income multiple. This relationship is consistent with both other researchers' findings as well as typical public (relatively large) company pretax operating income multiples of over twice that of our data size median of 4.51 times.[13] Extremely large privately held companies (say $500 million in value) are near-perfect substitutes for publicly traded stock in that the total cost to go public as a percent of its size becomes small.

Our conclusions are shown graphically in Figure 2.

After converting the above pretax operating income to pretax FCFF, we can convert the above to the implied cost of capital used with *pretax* income, with the results as shown in Figure 3.

Therefore, if we use a 24% cost of capital discount rate on the pretax unlevered FCFF of a typical ($4 million revenue) small privately held business that has a stable/normal margin in a discounted cash-flow model (DCF), we will derive an implied multiple of approximately 4.51 times pretax operating income. As you can see, our
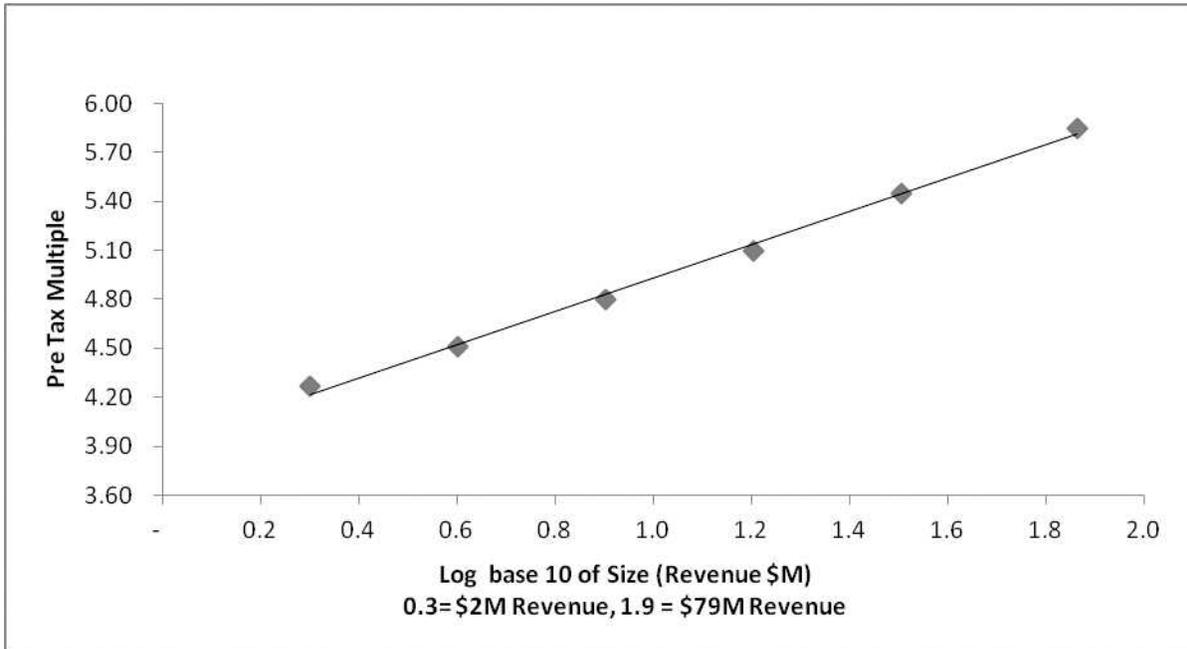
Figure 2
Normalized Market Multiples

methodology is, in essence, back solving for the reliable and observable cost of capital result.

However, unlike a completed transactions/market approach, we do not have to be concerned if the particular valuation assignment subject's growth outlook of FCFF is stable and/or comparable to the selected transaction multiples. With the DCF approach, we can, of course, forecast significant changes in FCFF. Therefore, we have the benefits of the completed transaction approach (direct interpolated comparability) without its limitations.

We also note that if the valuation assignment requires an analysis of after-tax income, the analyst simply needs to
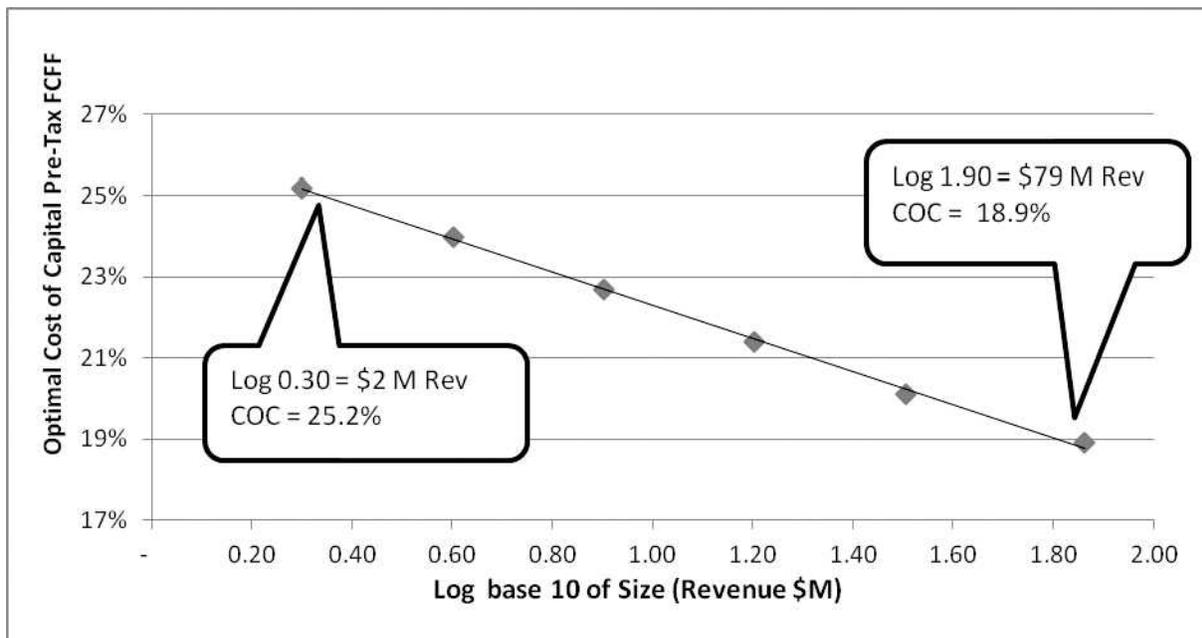

Figure 3
IPCPL - Size versus COC

Table 1
Hypothetical Transactions and Multiples

Trans #	Reported Mult	Trans #	Reported Mult	Trans #	Reported Mult
1	4.5	7	4.5	13	4.5
2	3.5	8	3.5	14	3.5
3	5.5	9	5.5	15	5.5
4	4.5	10	4.5	16	4.5
5	2.5	11	2.5	17	2.5
6	6.5	12	6.5	18	6.5
Mean	STDev				
4.5	1.35				

convert the cost of capital above by multiplying it by $(1 - \text{tax rate})$. Provided there is no abnormal long-run large aggregate difference between depreciation and capital expenditures, this after-tax approach should yield results nearly identical to the pretax design.

Model Data Set Criteria and COC Conversion Methodology

We utilized 830 qualifying completed transactions from Pratt's Stats and BIZCOMPS. Our criteria were as follows:

Pratt's Stats	BIZCOMPS
Revenue > 2 million; Owner compensation data provided Asset sale	Revenue > 2 million
Positive operating income	Positive SDE*
Transaction date 2000 through present	Transaction date 2000 through present
Seller and buyer equals private company	

 * SDE: sellers' discretionary earnings

The most common concern we encounter regarding these data is that they are inherently flawed by way of imperfections such as known examples of buyers paying crazy prices and/or incorrect reported information. Indeed, based on obvious transaction duplications between Pratt's Stats and BIZCOMPS, we can see that some of the data were occasionally contradictory by significant amounts. However, when we employ a large sample size, we can be confident that this noise is eliminated. Indeed, Toby Tatum, who has performed a great deal of related research, says:

> "If there is no intentional selection bias by the business brokers who supply these database developers with transaction data, then regardless of whether or not the acquisition's cash flow provides your opinion of a FMV salary and return on investment for the buyer, the databases provide an acceptably accurate indication of the real-world marketplace for the buying and selling of small businesses."

Data set reliability test:

In order to demonstrate the ability of our large sample size to cure the bad data problem, we performed a statistical analysis.

Let's assume that if there were no "crazy" prices paid by buyers or sold by sellers, and if the reported transaction data relevant to determining a price to operating income multiple were perfectly accurate, the "true" multiple

Table 2
Statistical Summary of Transactions

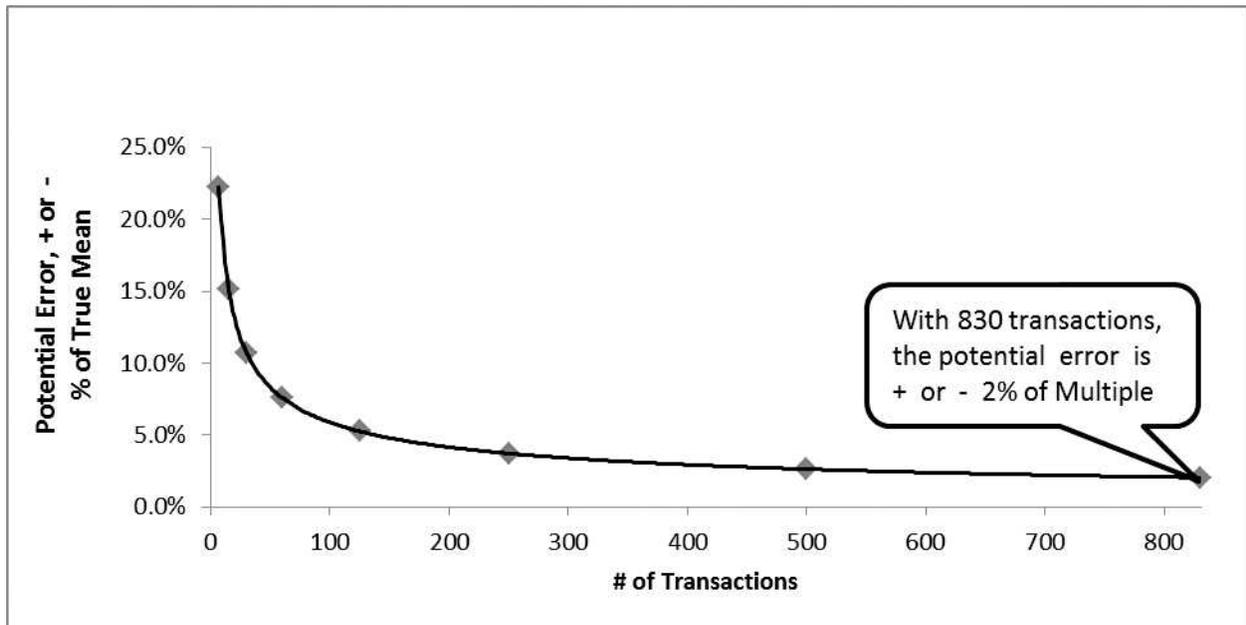| Group	Mult	R^2	No. Companies	Mult $\times$ No. Companies
Manufacturing	4.51	0.467	172	775
Retail	4.43	0.449	110	487
Distributors	4.66	0.438	128	597
Contractors	3.47	0.384	108	375
Restaurants	4.23	0.617	39	165
Other	4.20	0.423	273	1,147
			830	3,547
Conclusion (WTD)	4.27			

Figure 4
Reliability Analysis (95% Confidence Interval)

would always be 4.5×. Furthermore, let's assume a significantly large actual data problem, where only one of every three transactions is correct (4.5×), and the other two-thirds are badly distorted as shown in Table 1.

As we can see from the statistical analysis (Fig. 4), even assuming a very significant portion of the data is highly unreliable, our data set of 830 transactions is nearly perfectly reliable.[14] Specifically, with a sample size of 830, we are 95% confident that the reported data mean is between 4.41 and 4.59 if the true mean is 4.50.

Conversion from operating income to FCFF

Free cash flow to the firm (FCFF) is conceptually the steady-state amount of cash flow (after a reserve to replace the depreciable assets necessary for FCFF) available to distribute to shareholders and/or reinvest in growth opportunities.

Operating income is an accurate proxy for FCFF (provided depreciation is an accurate measure of a reserve for replacing the assets in place) if we assume a non-inflationary equilibrium.[15] Given these same assumptions, our proxy for pretax FCFF is operating income.

In a noninflationary equilibrium, margins are stable, and implied returns are equal to the cost of capital, and therefore any reinvestment of net income, by definition, is at a zero net present value (NPV). It is axiomatic that, under this set of assumptions, any *increase* in G by way of retained earnings is offset exactly by a reduction in the ratio of net income/purchase price.

Therefore, with the Gordon Growth Model, applying the unlevered assumptions to a *noninflationary*

equilibrium, the pretax cost of capital for a pass-through entity is:[16]

$$COC = Operating\ Income/Purchase\ Price$$

Pepperdine Model

John Paglia of Pepperdine has developed his own empirically based model to determine the COC by utilizing a survey of actual market participants with respect to their actual required return.

We specifically note that surveying data from the relevant market has been Pepperdine's founding premise. Unfortunately, they warn that for the vast majority of private companies (less than $5 million in market value), appraisers should not use their data:

> "Small businesses (those that don't qualify under any of the credit boxes in the survey) rely on a variety of financing sources that are not priced by institutional capital providers. As a result, the Pepperdine cost of capital survey does not have market-driven empirical data at this time to support discount rates for this segment of the economy."

We tested our results against Pepperdine's survey. After adjusting the Pepperdine data for leverage using the Hamada Model, our results were similar:[17]

Pepperdine survey model (unlevered)

$$cost\ of\ equity = 22.8\%$$

$$IPCPL = 18.7\%$$

Implications for CAPM, BUM, and Modern Portfolio Theory

Given the clear statistically significant and highly inverse relationship between size and valuation multiples for privately held companies, we are duty-bound to address the cause. Some may be inclined to discount this relationship as one that simply mirrors the small stock premium. However, most researchers in this area believe that for publicly traded stocks, size is just a proxy for other systematic risk factors. Given that CAPM is supposed to measure systematic risk and the small stock premium is measured after the appropriate CAPM adjustment, most researchers believe CAPM is mis-specified. Most believe that CAPM has an ''intertemporal'' flaw that causes it to underestimate the systematic risk of small companies, which have much lower margins and consequently are far more susceptible to systematic revenue shocks from recessions (please see Appendix B). Even Duff & Phelps now acknowledges and demonstrates both the *observable relationship of operating margins and risk/return, as well as the need to adjust CAPM upward for lower operating margins.*

However, because our analysis of size differences controls for differences in operating margins and related intertemporal-based beta flaws, we see no systematic risk differences. Therefore, unlike small publicly traded stocks, the significant size premium we observe here is contrary to both CAPM *and* MPT. The relationship we document is consistent with typical COC/BUM premiums for ''company-specific risk'' factors such as:

- Lack of product diversification
- Lack of geographic diversification
- Lack of customer diversification

There is a clear logical correlation between these factors and size; as size decreases, the company-specific risks generally increase. Larger private companies generally have more products, operate in more regions, and have more customers.[18] If MPT is strictly applied, none of this would matter. *Since we can observe that nearly all small privately held business owners are extremely undiversified, we believe this to be an excellent starting place to make adjustments to the model.*[19]

With the state-of-the-art/BUM, completely subjective upward adjustments to the cost of capital are typically in the range of 7% to 10%. With the IPCPL, this net upward adjustment is already built in. The average adjustments for size, company-specific risk, systematic risk, and liquidity/finance ability are, by definition, *zero*. This way, subjective adjustments for CSR must be relative to more similar small privately held businesses of the same size that already expose owners to

dramatically higher levels of the aforementioned list of negatives. This is a far better result than previous state-of-the-art methods resulting in large subjective adjustments.

Although we do not attempt to prove a theoretical model that could duplicate the results in this paper, we note importantly that *MPT strictly applied does not work for small privately held businesses.*

Conclusion

We analyzed the assumptions of MPT and demonstrated qualitatively and quantitatively the pitfalls of assuming that the assumptions of MPT hold for small privately held companies. Indeed, we demonstrated how typical ownership of privately held companies generally violates strictly applied MPT and, by extension, demonstrated the lack of comparability between publicly traded equity securities and small privately held businesses. Since our industry's current state-of-the-art cost of capital approaches utilize the assumptions of MPT, as well as the returns from these same incomparable equity security returns, as its fundamental starting point, the extrapolation to small privately held businesses was demonstrated to be unreliable. Therefore, we developed the IPCPL, which solves all of the following major incomparability problems:[20]

- Company-specific, aka unsystematic, risk
- Transaction costs
- Liquidity
- Alleged ''small'' stock premium
- Taxes
- Leverage

The entire list of incomparability problems is due to utilizing publicly traded equity security returns as the cost of capital model's fundamental starting point. We showed how to construct a cost of capital model that has as its fundamental starting point estimated ex ante returns derived from actual market clearing prices from *actually comparable* small privately held businesses.

Now we can see the tie-in between the economist joke and the current state of the art. With the IPCPL, we now have a light for the dark alley, and in our next paper, we will better focus that light and show how to better estimate the cost of capital for a specific company. In short, we believe that, ''In the land of the blind [the current state-of-the-art that utilizes publicly traded equity securities' returns], the one-eyed man [in this case, an appraiser who does not have to worry about inherent risk, size, tax, liquidity, and leverage differences] is king.''

Endnotes

1. For our purposes, we define this as privately held businesses worth $5 million or less.
2. Toby Tatum, ''A New Method for Building a CAPM Discount Rate for Small Businesses Based on SBBI Data,'' Business Appraisal Practice (Third Quarter 2010), with permission.
3. One of the authors of the Pepperdine Private Capital Markets Surveys in a LinkedIn discussion forum.
4. In a recent Pepperdine survey, 78% of respondents did not feel comfortable with our industry's current cost of capital methods, using returns on publicly traded equity securities (*Pepperdine Private Capital Markets Project, Survey Report III*, Summer 2010).
5. Court of Chancery of Delaware, New Castle County 902 A.2d 1130 (2006).
6. We use company-specific risk (CSR) interchangeably with uncorrelated risk, aka idiosyncratic, unsystematic, and diversifiable risk. Duff & Phelps in their latest annual cost of capital literature has acknowledged the confusion surrounding their company-specific risk adjustments and that their models do not account for unsystematic risk and their models' returns are on a ''publicly traded equivalent'' basis.
7. This difference, of course, becomes larger (smaller) as the size of the privately held business becomes smaller (larger).
8. Due to the economic concept of ''the diminishing marginal utility of money,'' it would be irrational for anyone not to be risk averse. Assume a person was given $50,000 and had to acquire things with it and was subsequently and unexpectedly given another $50,000 and had to acquire things with it. When asked which of the two lots of things is worth more to them, people naturally say the first one, and, therefore, we can see that wealth/buying power has a diminishing value and, therefore by extension, losing net worth costs more than the offset of gaining net worth.
9. Hui Chen and Jianjun Mio, ''Entrepreneurial Finance and Non-Diversifiable Risk,'' accessed at econ.as.nyu.edu/docs/IO/9299/083w-chen.pdf, 8 December 2008, Oxford Journals, Oxford University Press, 2010.
10. Vasia Panousi and Dimitris Papanikolaou, ''Investment, Idiosyncratic Risk, and Ownership,'' accessed at http://www.federalreserve.gov/pubs/feds/2011/201154/201154abs.html, 2 December 2011.
11. We encourage the reader to see Appendix A (FAQ) for the details of our methodology.
12. ''Normalized/stable'' is the operating margin that implies competitive equilibrium, where margins are as likely to increase as they are to decrease.
13. Pepperdine's survey shows that as size increases, the cost of capital decreases (*Pepperdine Private Capital Markets Project, Survey Report III*, Summer 2010). Also see Jerry O. Peters, ''Adjusting Price/Earnings Ratios for Differences in Company Size,'' Business Valuation Review (June 1999):71–85; Toby Tatum, Institute of Business Appraisers' Business Appraisal Practice ''Revisiting the Size Effect Phenomenon for Small Businesses, 1st quarter 2012 pp 7–8.'' Also Mergerstat Review & GF Data concluded a large size premium; see http://www.acg.org/UserFiles/file/global/Press%20Releases/GF%20Data%20Resources%20Q2%20Draft%208%2023.pdf.
14. Although we are not aware of any research that claims that these data providers' transaction data are *systematically* biased (net net), we note importantly that we must qualify our confidence interval claims accordingly.
15. See Appendix A.
16. We note that the purchase price for the asset sale is, of course, independent of the seller's tax status. Operating income is defined here as net income plus interest and federal income taxes.
17. Based on John Paglia's Pepperdine handout at ASA 2011 advanced BV seminar in Chicago. Based on a midpoint of 30% COE points for pretax income ($5 million EBITDA, $25 million enterprise value @100% D/E per example). Hamada unlevered beta = 1.2, ERP 6.0, tax rate = 0.
18. Firm size is also positively correlated with debt capacity as a percent of capital. Smaller firms with higher CSR have higher default risks at similar debt to capital percentages.
19. We note that a great deal of recent research has found strong positive correlations between liquidity/marketability discounts and company-specific risk. Naturally, size-relative differences in correlated/systematic risk measured via the Fama French Three Factor Model and/or CAPM would be a logical basis for adjustment as well.
20. Please refer back to the list of problems in the introduction for a more detailed explanation of the specific problems.
21. Source for S&P data is Damodaran web page. At http://pages.sternnyu.edu/~adamodar/.
22. Banz, Rolf W. ''The Relationship Between Return and Market Value of Common Stocks,'' *Journal of Financial Economics* 9 (1981):3–18.
23. John Y. Campbell and Tuomo Vuolteenaho, The National Bureau of Economic Research (NBER) Working Paper No. 9509, issued in February 2003.

24. Eugene F. Fama and Kenneth R. French, ''Size, Value, and Momentum in International Stock Returns,'' Fama-Miller Working Paper; Tuck School of Business Working Paper No. 2011-85; Chicago Booth Research Paper No. 11-10; accessed at: http://ssrn.com/abstract=1720139, 21 June 2011. SMB = s̲mall company return m̲inus b̲ig company return and HML = h̲igh book to price m̲inus l̲ow book to price.

Appendix A: Frequently Asked Questions

- How did you handle owner's compensation?
  BIZCOMPS' ''SDE'' is reported pre-owner's compensation. In Pratt's Stats, we selected only the transactions that reported owner's compensation. We then used market compensation surveys to adjust owner's compensation. For all 830, we added back actual compensation and subtracted market compensation.
- What was the minimum revenue?
  We selected all qualifying transactions above $2 million in revenue. Below $2 million, we determined that the potential estimation error regarding fair market compensation of the owner-operator would be too significant as a percentage of operating income.
- Did you include stock transactions?
  No. The universe of transactions (approximately 90%) was primarily asset transactions anyway. We believe that asset sales, all else being equal, have higher values due to higher tax shields. Therefore, we excluded stock sales.
- How did you adjust for the historical reported income lag and the fact that the Gordon Growth Model's FCFF is based on a forward FCFF?
  Pratt's Stats reports the dates for the latest income data and the transaction date. We adjusted income up by a 3% annualized rate to gross it up to the transaction date. We used the same relationship to adjust BIZCOMPS to the BIZCOMPS transaction date. (Note: an adjustment of income by 3% for one year only impacts the *ratio* of price to income [return] by 0.8%.) We then grossed up the income by another 3% to get to a pro forma income. We then reduced this pro forma operating income figure by 3% to reflect the fact that inflationary operating income growth consumes working capital and inflationary capital expenditures (over depreciation) to derive $FCFF_1$. Depreciation for BIZCOMPS was estimated by using 10% of the reported FF&E market value.

- Did you include any of the completed transactions where the buyer was a public company?
  No. We believe this may introduce a potential ''observation bias'' in the figures. These companies may indeed be companies that have some high net present values of growth capital. By observation bias, we are referring to the fact that the proportion of firms in the databases acquired by a public company is many times the proportion of all comparably small private firms that could be acquired by a public company. In other words, the uniqueness of the firm makes it more likely to be acquired by a public company and show up in the databases. This bias is not necessarily a problem in that as long as we can estimate G, it, of course, is of no consequence if the buyer was a public company. We believe that our generalization of greater G being offset by a lower ratio of P/FCFF is reliable as long as a substantial number outliers that have a very special ability to earn very substantial excess profits are excluded. We believe that small private companies that are acquired by a public company are disproportionately likely to be those special companies.
- Is your observed cost of equity levered?
  No. Since we solve for COC based on control basis P and unlevered FCFF derived from adjustments to unlevered operating income, we are solving for optimal COC, and since P is the control price, it is also the *optimal COC*. This way, we can use the model where we do not need (want) to make any assumptions regarding debt leverage or interest rates on debt. We recommend that users employ the model unlevered and then simply subtract the market value of any interest-bearing debt to derive the value of equity.
- How did you account for the fact that the observed reported transactions exclude cash and (noninventory) operating working capital, in the case of Pratt's Stats (assuming an asset sale), and also exclude inventory in BIZCOMPS?
  We assumed that the buyer on average would need to make an additional investment above the reported deal price equal to 5% of revenue and adjusted the overall P/FCFF ratio accordingly.
- How did you incorporate transactions where the reported operating margins were significantly below or above normal and therefore expected to change?
  We excluded all completed transactions that had negative FCFF. We normalized the calculated P/FCFF ratio with a multiple regression model. Since

subnormal or extra-normal margins affect the price ratio in a nonlinear way, we segregated the data into four groups—two for below-average margins and two for above-average margins. This way the ''constant'' in the regression model is the normalized multiple. If the data had no noise, we could simply observe the FCFF to price ratio for firms with nearly exactly normal margins. Since the completed transaction multiple was highly correlated with margin, we could normalize the margin in the regression in a way where the model is solving for the regression ''constant.'' Had the operating margin been normal, the FCFF to P ratio would have been predicted to be ''constant.'' The standard error of constant and the model in general was not significant due to the very large 830 transaction sample.

- Why did you normalize margin effects around the mean instead of the median?
  We calculated the implied cash flow return on investment (CFROI) internal rate of return (IRR) on investment including all intangibles at estimated replacement cost) at the median and the mean and found that the CFROI was below the implied COC at the median. At the mean, it was nearly identical. Also, the implied COC was not significantly sensitive to increases *above* our selected breakpoint.

- Why didn't you just take the average or median multiple instead of estimating the normalized ratio?
  The relationship of margins to the ratio of FCFF/P is not linear. We want to know that ratio when the buyer and seller (830 transactions or 1,660 votes) believe it is stable.

- How did you adjust the P/FCFF multiple for size?
  We calculated the (log) of revenue and the difference of the log of revenue from the approximate \$4 million median revenue size for each of our six groups. Therefore, a zero figure for this independent variable would be for the predicted constant at the median size point. The regression coefficient for size was positive in all six groups (larger-sized businesses in the same group had higher multiples) and was significant at the 90% level for five of the six. We averaged the coefficient from all six groups in our generalized COE model.

- What was the R^2 of your model?
  The R^2 values of each of our multiple regression models are shown in Table 2 (data in the table are before our overall working capital, including required cash, transaction price adjustment).

- It seems like your model is simply backing into the COC that would yield values equal to normalized completed transaction comparables as applied to the subject. Is that cheating?
  No, it's not cheating; however, your characterization is a good one, except we doubt anyone could adjust for size as accurately as our model.

- What if significant numbers of the targets in the completed transactions were growing at a significant rate?
  Based on our experience, we estimate for businesses in this size group (\$4 million median) that the vast majority have capital expenditures (CAPX) that is either below normalized depreciation or insignificantly higher. They are merely spending money to replace the wear and tear of the existing business. Therefore, FCFF will grow at the rate of inflation for most all businesses we measured. Therefore, based on this and other observations, we can conclude that this is not a significant issue.

- What about the minority of businesses that are growing at a substantial rate?
  So, G can be higher than inflation if the company expands. However, if the net present value of the expansion is zero, the growth CAPX G improvement will, by definition, have an offsetting cost on yield (FCFF/P).

- How do we know that the net present value of the expansion capital requirement will be near zero for the minority of firms that are growing?
  We utilized the economic concept of zero economic profits on average. We also tested the zero economic profits concept and found confirmation as follows: For the period 1960–2010, the S&P 500 had earnings of \$3.10 in 1960 and \$83.70 in 2010.[21] Inflation for this period was a geometric average of 3.99%. The total geometric growth of earnings was 9.07%. Therefore, excluding the earnings growth of inflation that would occur without reinvestment of income, the earnings growth rate is only 5.08%. Given an average earnings retention rate of 47% for the period and a 10% cost of equity, we would expect approximately a rate of growth over inflation from retained earnings of approximately 4.7%. A comparison of the 4.7% expected rate to the actual rate of 5.08% indicates that the S&P 500's return on incremental capital was only slightly higher than its cost of capital. Therefore, we have relevant, broad, and long-term empirical evidence that our assumption regarding G is reasonable. Of course, we do not imply that all firms cannot add incremental net present value, only that on average it is not meaningfully large. This empirical observation is consistent with the economic theory that ''physical'' (investments in

projects)/nonfinancial capital markets are roughly efficient on net in general.

Based on the reasonable application of efficient market theory and empirical research we assume that assumptions of the value of G are consistent with a zero net present value of incremental real expansion capital.

- What period of time do you use for your completed transactions data?

  We use ten years of data. We adjusted the normalized multiple for the current expected return on the market. For example, if Professor Damodaran's implied ERP for the last ten years was 4.4% and his current rate is 5.5%, we adjust the COC model up by 1.1% (and the operating income multiple down from aproximately 5.0). In other words, our model is roughly analogous to the small stock premium method, provided you were updating your conditional equity risk premium (ERP) or using an implied model like Professor Damodaran recommends.

- Could we look at your regression models?

  Yes, our model can be downloaded at biz-app-solutions.com and we hope others will adopt this approach and post their results.

Appendix B: The Small Stock Premium

Our profession extensively applies the small stock premium from public stock markets to subject companies. Therefore, understanding the cause of the small stock premium would assist us in the determination of the appropriateness or inappropriateness of using the small stock premium in our valuations. Many seem to believe the small stock premium makes sense intuitively ''because small stocks are more risky.'' Before we can investigate the cause of the small stock premium, however, we must understand what risk is.

The usefulness of reducing risk through diversification, which is the concept behind ''modern portfolio theory'' (MPT), is an extremely important component in risk analysis. Using this theory, uncorrelated risk, also known as diversifiable or company-specific risk (CSR), is generally thought of to be of no consequence in well-diversified portfolios. The cost of complete diversification in public stock markets can be as little as $7.00 to buy an exchange-traded fund (ETF) made up of hundreds of individual stocks.

Consequently, taking on uncorrelated risk in the public markets is not rewarded in terms of higher return. We will now compare correlated or undiversifiable risk with unsystematic risk. Systematic risk cannot be eliminated in a portfolio of stocks and is related to the future unknown strength or weakness of the economy. Companies that are

highly dependent on the strength of the economy, homebuilders, for example, have very high correlated risk. One can add all the homebuilders one wants to a portfolio to eliminate the CSR, but individual homebuilder companies will all correlate together with changes in the economy's outlook. Our current cost of capital practices use CAPM beta to measure and account for this correlated risk. An undiversified small privately held business owner is, of course, also confronted with this type of risk. Provided beta is an accurate measure of correlated risk, our current business valuation practices correctly price this risk.

The small stock premium is typically measured after considering beta. Therefore, we need to be careful with our terminology. If traditional MPT holds, the small stock premium should not exist, in that diversifiable/CSR is not rewarded due to the ease of eliminating it through diversification, and the CAPM/beta is supposed to account for undiversifiable or correlated risk.

So, what then is the cause of the small stock premium? Also, if we don't know its cause, how can we extrapolate it to our appraisals?

Rolf W. Banz is credited with the discovery of the small stock premium. He stated:[22]

> ''This 'size effect' has been in existence for at least forty years and is evidence that the capital asset pricing model is misspecified. The size effect is not linear in the market value; the main effect occurs for very small firms, while there is little difference in return between average sized and large firms. It is not known whether size per se is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size.''

Since CSR is correlated with size, is it unsystematic risk?

Theory, common sense, and empirical research say no.

If CSR were rewarded with higher returns, we would witness an explosion of ETFs that would buy hundreds of stocks with high levels of company-specific risk, thereby eliminating the company-specific risk premium in the future. Therefore, based on common sense and MPT, company-specific risk is not a cause of the small stock premium.

What about empirical research?

Richard Roll and Stephen A. Ross wrote ''An Empirical Investigation of the Arbitrage Pricing Theory'' in the *Journal of Finance* in December 1980. In this study, they found that company-specific risk (variance of return) was not priced in stock returns when measured together with (three to four) systematic risk factors. Indeed, they admitted that the arbtrage pricing theory (APT) model would have to be rejected if individual stock variance (''own'' standard deviation) of stock returns did cause

higher returns, because it would violate the efficient market hypothesis on which the APT is based.

Moreover, the most commonly accepted cost of equity (COE) model in academia is the Fama French Three Factor Model. This model is based on extensive research that found that stock returns are best correlated with three factors. One of the factors is the size of the company. Even though CSR is highly correlated with stock returns, they do not include CSR as a factor. This is because with the inclusion of more than one factor in a statistical model, a variable such as CSR loses it predictive power— correlation does not prove causation. CSR ''covaries'' with other factors, and some other factors correlate better than CSR, so it is excluded from the model, similar to the Ross and Roll study conclusion.

On the other hand, a few researchers have found that CSR is correlated with excess returns (over CAPM) and suggest causation. However, any attempt to scientifically suggest causation must be in a multiple regression format that includes simultaneously other variables that also correlate with the excess returns.

So where does this leave us?

The most prevalent theory regarding the cause of small stock premium is based on the ''intertemporal problem,'' which suggests that CAPM is severely flawed in that it underestimates the true correlated risk of small stocks. This is a very hot area of finance with a lot of ongoing research. For example, Campbell and Vuolteenaho[23] have an excellent analysis of the phenomenon in their paper ''Bad Beta, Good Beta'':

> ''This paper explains the size and value ''anomalies'' in stock returns using an economically motivated two-beta model. We break the CAPM beta of a stock with the market portfolio into two components, one reflecting news about the market's future cash flows and one reflecting news about the market's discount rates. Intertemporal asset pricing theory suggests that the former should have a higher price of risk; thus beta, like cholesterol, comes in ''bad'' and ''good'' varieties. Empirically, we find that value stocks and small stocks have considerably higher cash-flow betas than growth stocks and large stocks, and this can explain their higher average returns. The poor performance of the CAPM since 1963 is explained by the fact that growth stocks and high-past-beta stocks have predominantly good betas with low risk prices.''

The Duff & Phelps (D&P) *Risk Premium Report* provides some very useful information. D&P shows how risk premiums correlate with size and operating margins. All other things being equal, cash flow risk will be higher for companies with lower margins. Given the description in ''Bad Beta, Good Beta'' regarding ''cash flow betas,'' we can see the causal link to higher returns for lower-margin

companies. However, D&P falsely attributes this to company-specific risk. Others have studied earnings betas and found strong inverse relationships between earnings betas and firm size.

Are differences in liquidity partly responsible? Fama and French point to facts that strongly imply liquidity is not the primary cause:[24]

> ''For a liquidity story to work, small growth stocks and small momentum losers must be more liquid and/or have lower sensitivity to liquidity factors than small value stocks and small momentum winners. The results of Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) suggest that this is unlikely.''(footnote)

Is the small stock premium declining? Actually, maybe it is disappearing. Fama and French state the following:[24]

> ''There is no size premium in any region during our sample period. Average small minus big (SMB) returns are all close to zero (Table 1). In contrast, there are value premiums in all regions. Average high minus low (HML) returns range from 0.33% per month (t = 1.48) for North America to 0.62% (t = 3.04) for Asia Pacific. As in the U.S. results of Fama and French (1993), Kothari, Shanken and Sloan (1995), and Loughran (1995), value premiums are larger for small stocks.''

Interestingly, the small stock premiums for Europe, Japan, and Asia were all negative for the twenty-year period. However, they, like the positive U.S. premium, were also not statistically significant.

Moreover, what does the evidence suggest about the standard error of the estimate of the size premium?

We observe that, for the period 1963–2010, the average annual small stock premium was 6.4%, but the standard deviation of the premium over the same period was a very large 26.3%. Therefore, even if we assume that the 1963–2010 period perfectly represents the future, the actual premium at the 90% confidence level is somewhere between 0.2% to 12.6% (an extremely large range).

What are the implications of using these unreliable data for predicting future returns for a different asset class?

An underlying assumption of the existing small stock premium evidence is that the past forty-eight years reasonably represent the future. Since the cause of the highly variable small stock premium is uncertain, our ability to assess its magnitude and its importance is challenged at best.

In summary, the amount and cause of the small stock premium is uncertain, and most believe it is due to higher earnings betas caused by lower margins, and therefore extrapolating size to our subject small privately held businesses will result in large errors, given the substantially different operating margins of our subject companies.